

# Low Complexity Channel Estimation for Multi-user Massive MIMO Systems with Pilot Contamination

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**Abstract - The massive use of transmit and receive antennas with perfect channel state information (CSI) in massive multiple-input multiple-output (mMIMO) systems can lead array power gain increments proportional to the number of antennas. For CSI estimation pilots can be used, but in multi-user scenarios pilot contamination may occur which may compromise estimation, even when are used orthogonal sequences such as Chu sequences. In such conditions improvements achieved by mMIMO systems can be compromised. Therefore, it is important to assure good channel estimation even when pilot contamination exists without sacrificing spectral efficiency and complexity. For that purpose, methods such as a complexity-reduced adaptive semi-blind channel estimator or a decision directed scheme based on a iterative block frequency domain equalizer (IB-DFE) could be used to compensate the pilot contamination impact on channel estimation. Moreover, when the the number of channels involved in communication the first method has comparable performance to second one, for a lower complexity which could. To support this assumption it is also presented a set of performance results comparing both methods.**

**Index Terms: massive MIMO, Channel state information, decision directed channel estimation, pilot contamination.**

## I. INTRODUCTION

Last years have witnessed an exponential increase on demand of high rate wireless transmissions due to the penetration of smart phones. To fulfil this demand massive multiple-input multiple-output (mMIMO) could be employed to increase system capacity [1], [2]. For that reason mMIMO has attracted considerable attention as a possible key technologies in the forthcoming fifth generation (5G) of wireless communications [3]. Given perfect channel state information (CSI), the signals received at all antenna elements can be combined coherently and the array gain grows without bound with the number of antennas at the access point [4]. Under these conditions the massive use of antennas elements can overcome both multiuser interference and thermal noise for any given number of users and any given powers of the interfering users. Note that this requires CSI available at both ends of the link. For that reason channel estimation has an important role in mMIMO transmission.

Under ideal conditions the training sequences should be uncorrelated to avoid pilot contamination, which can be granted by orthogonal sequences [5], [6], [7]. For that purpose sequences with periodic auto-correlation function such as Zadoff-Chu (ZC) sequences can be used [7], without the need of any pre-coding which may reduce complexity. However, pilot contamination arises by the repeated use of same training sequences by several users. Thus, becomes crucial for mMIMO systems to develop estimators capable of accurately estimate CSI with fewer pilots than traditional pilot-based channel estimation techniques. Hence, semi-blind channel estimators are effective in mitigate the effect of pilot contamination, as shown in [8] and [9]. These latter estimators are based in eigenvalue decomposition (EVD) algorithms and few pilots are used to resolve the problem of the ambiguity matrix. Another method to resolve the ambiguity problem is to exploit the asymptotic orthogonality of the users' channel, supported by the law of large numbers. This last method is commonly implemented with singular value decomposition (SVD) and achieves better estimation results than EVD based estimators [10]. However, the main impediment of both SVD and EVD-based channel estimation techniques emanates from the computational complexity  $O(N_R^3)$ , proportional to the dimension of the received signal, which makes them not recommendable for truly mMIMO scenarios with a very high number of antennas. Having in mind these considerations pilot contamination compensation and low complexity in channel estimation should be assured by estimation methods. Low complexity in channel estimation under pilot contamination conditions could be assured by methods such as a complexity-reduced adaptive semi-blind channel estimator or a decision directed scheme based on a iterative block frequency domain equalizer (IB-DFE).

In this paper, two channel estimation methods will be described and compared. They require fewer pilots to estimate the CSI, relieving the effect of pilot contamination and increasing spectral efficiency. The first one consists in an an iterative channel estimation technique based on the outputs of an IB-DFE [11], [12], [13], [14] already analyzed in [15]. One advantage of this method lies on the mitigation of the effects resulting from pilot contamination or in a pilot number reduction. The second method named fast single compensation approximated power iteration (FSCAPI) is a complexity-reduced adaptive semi-blind channel estimator that uses a subspace tracking algorithm and cloud achieve higher

estimation speeds with good tracking performance [16], [17]. After this introductory part the rest of the paper is organized as follows: systems characterization is presented in section II. Section III, characterizes the proposed technique. Simulation results of both methods regarding scenarios without and with pilot contamination are presented and compared in section IV. Conclusions are drawn in section V.

## II. SYSTEM CHARACTERIZATION

The system adopts a time division duplexing (TDD) transmission with  $N_u$  users, each one with  $N_a$  antennas at user's terminal (UT) and  $N_R \geq N_u$  receiving antennas at the base station (BS). It is adopted a single carrier with frequency domain equalization (SC-FDE) modulation. It is also assumed channel reciprocity between uplink and downlink. In downlink a block diagonalization (BD) method [18], [19] is used to cancel co-channel interference (CCI). Under these conditions results for the uplink a  $N_R \times N_u$  MIMO configuration for each user  $u$ , through a channel  $\mathbf{H}^{UL}$  ( $\mathbf{H}^{UL} \in \mathbb{C}^{N_R \times N_u}$ ), as shown in Fig 1. The elements of  $\mathbf{H}^{UL}$  are samples of independent and identically distributed (i.i.d.) complex Gaussian process.

On the other hand, it is well known that an IB-DFE receiver can deal with the channel selectivity and reduce interference [13], [14]. The  $u$ th UT transmits the block of  $N$  data symbols  $\{x_n^{(t)}; n = 0, 1, \dots, N-1\}$  being  $\{y_k^{(r)}; k = 0, 1, \dots, N-1\}$  the received block at the  $r$ th BS antenna (as with other SC-FDE schemes, a cyclic prefix is appended to each transmitted block and removed at the receiver).

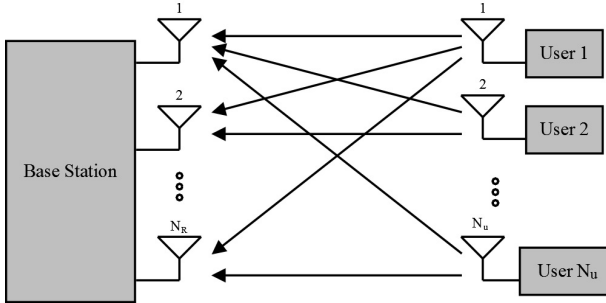


Fig. 1. Uplink MU-MIMO system model with single-antenna users

By adding a cyclic prefix that corresponds to a periodic extension of the useful part of the block, i.e.,  $s_{-n} = s_{N-n}$  with a length higher than the overall channel impulse response, null inter block interference (IBI) can be achieved. Since at receiver the samples associated to the cyclic prefix are discarded, the impact of a frequency-dispersive channel is equivalent to a scaling factor affecting each frequency. Under these conditions the corresponding frequency domain received signal is given by

$$\mathbf{Y} = \mathbf{H}^{UL} \mathbf{X} + \mathbf{Z}, \quad (1)$$

where  $\mathbf{Y}$  ( $\mathbf{Y} \in \mathbb{C}^{N_R}$ ) is the received at the base station ( $nr = 1, 2, \dots, N_r$ ),  $\mathbf{X}$  ( $\mathbf{X} \in \mathbb{C}^{N_u}$ ) is the transmitted signal by the  $N_u$  UT, and  $\mathbf{Z}$  ( $\mathbf{z} \in \mathbb{C}^{N_R}$ ) is the array relative to additive white gaussian noise (AWGN) with zero mean and variance  $\sigma_z^2 = \frac{N_0}{2}$ , where  $N_0$  is the noise power spectral density. The channel matrix  $\mathbf{H}^{UL}$  contains all single input single

output (SISO) channel responses from each BS transmitting antenna to each user's receiving antenna. The elements of  $\mathbf{H}^{UL}$  are samples of independent and identically distributed (i.i.d.) complex Gaussian process, given by

$$\mathbf{H}^{UL} = \begin{bmatrix} H_{1,1} & \dots & H_{1,N_R} \\ \vdots & \ddots & \vdots \\ H_{N_u,1} & \dots & H_{N_u,N_R} \end{bmatrix}. \quad (2)$$

The channel considered is a fading channel constant during the coherence time interval of length  $N$ , given by

$$h_{nr,nu}(t) = \alpha(t) e^{j\theta(t)}, \quad \forall nr, nu, \quad (3)$$

where  $\alpha(t)$  denotes the envelope and  $\theta(t)$  represents the phase of the equivalent channel response.

### A. Multi User MIMO

In the downlink of MU-m-MIMO systems precoding methods are used to eliminate interference between antennas, assuming that we have perfect CSI. In these scenarios, block diagonalization technique can applied in precoding schemes using SVD [20], [8], [9]. The precoded signal from the  $u$ -th user can be written as

$$\mathbf{X}_u = \mathbf{W}_u \tilde{\mathbf{X}}_u, \quad u = 1, 2, \dots, N_u, \quad (4)$$

where  $\mathbf{X}_u$  ( $\mathbf{X}_u \in \mathbb{C}^{N_u}$ ) is the array of transmitted symbols,  $\mathbf{W}_u$  ( $\mathbf{W}_u \in \mathbb{C}^{N_R \times N_a}$ ) the precoding matrix and  $\tilde{\mathbf{X}}_u$  the data symbol array concerning the  $u$  user. The data array received at BS can be described as

$$\mathbf{Y}_u = \mathbf{H}_u^{DL} \sum_{k=1}^{N_R} \mathbf{W}_k \tilde{\mathbf{X}}_k + \mathbf{z}_u, \quad u = 1, 2, \dots, N_u, \quad (5)$$

where  $\mathbf{z}_u$  is the term due to AWGN. When  $\mathbf{H}_u^{DL} \mathbf{W}_k \neq 0_{N_R \times N_R}$ ,  $\forall u \neq k$  there is CCI, where  $0_{N_R \times N_R}$  is a zero matrix. Null CCI is achieved as long as the effective channel matrix is block-diagonalized, i.e.,  $\mathbf{H}_u^{DL} \mathbf{W}_k = 0_{N_R \times N_R}$ ,  $\forall u \neq k$ . However, with pilot contamination we may have  $\tilde{\mathbf{H}}_u^{DL} \mathbf{W}_u \neq 0$ . On the other hand, at uplink pilot contamination compromises the CSI estimation, which may affect system performance. These effects may be compensated by channel estimation methods assuring null CCI for the remainder data blocks transmitted during the coherence time.

## III. CHANNEL ESTIMATION

ZC sequences, also called constant amplitude zero autocorrelation (CAZAC) sequences, are perfect polyphase sequences [7]. A comb type format is adopted for the arrangement of pilots [21] and it is assumed that channel conditions remain the same over several data blocks. Training sequences are inserted in the first  $N_P$  subcarriers of the corresponding frequency domain block  $X_k; k = 0, 1, \dots, N-1$  and are used to estimate the frequency channel response, which is used in equalization for the next  $N_{data}$  subcarriers ( $N_{data} = N - N_P$ ).

Let assume  $\mathbf{S}$  as the matrix containing the transmitted training sequences  $\mathbf{s}_{nu}$  ( $nu = 1, 2, \dots, N_u$ ). The received training sequences,  $\mathbf{Y}^P$  ( $\mathbf{Y}^P \in \mathbb{C}^{N_R \times N_P}$ ), are given by

$$\mathbf{Y}^P = \mathbf{H}\mathbf{S} + \mathbf{Z}^P, \quad (6)$$

where the pilot matrix,  $\mathbf{S}$  ( $\mathbf{S} \in \mathbb{C}^{N_u \times N_P}$ ), corresponds to the collectively transmitted training sequence by  $N_u$  users and  $\mathbf{Z}^P$  ( $\mathbf{Z}^P \in \mathbb{C}^{N_R \times N_P}$ ) is the AWGN noise matrix, which lines have the same characteristic as  $\mathbf{z}$ . Thus, at the BS, the signal received by the  $nr$ -th antenna ( $nr = 1, 2, \dots, N_R$ ), in frequency domain can be described by

$$\mathbf{Y}_{nr}^P = \mathbf{S}_1 H_{nr,1} + \mathbf{S}_2 H_{nr,2} + \dots + \mathbf{S}_{nr} H_{nr,nu} + \dots + \mathbf{S}_{N_u} H_{nr,N_u}, \quad (7)$$

where  $H_{nr,nu}$  ( $nu = nr$ ) is associated to the desired channel while the other terms  $H_{nr,nu}$  ( $nt \neq nr$ ) are due interfering channels. Interference is canceled by doing the Hermitian inner product operation in  $\mathbf{Y}_{nr}^P$  with the appropriate training sequence  $\mathbf{S}_{nr}$ . As a result the channel frequency response estimate  $\hat{H}_{nr,nr}$ , can be obtained through

$$\hat{H}_{nr,nr} = \frac{\langle \mathbf{Y}_{nr}^P, \mathbf{S}_{nr} \rangle}{N_{ZC}}. \quad (8)$$

When pilot contamination exists (in Fig. 2 is shown how the effect caused by the reuse of training sequences from other users in neighboring cells is modeled) the received pilots are given by

$$\mathbf{Y}^{PC} = \mathbf{Y}^P + c(\mathbf{H}^{PC}\mathbf{S} + \mathbf{Z}^{PC}), \quad (9)$$

where  $c$  ( $0 < c < 1$ ) is the attenuation constant that adjusts the power of the interference training sequences sent by  $K$  interfering users,  $\mathbf{H}^{PC}$  ( $\mathbf{H}^{PC} \in \mathbb{C}^{N_R \times K}$ ) denotes the interfering channel,  $\mathbf{S}^{PC}$  ( $\mathbf{S}^{PC} \in \mathbb{C}^{K \times N_P}$ ) is the interfering pilot matrix with  $K$  interfering training sequences (equal to the number of interfering as users) and  $\mathbf{Z}^{PC}$  ( $\mathbf{Z}^{PC} \in \mathbb{C}^{N_R \times N_P}$ ) is the correspondent interfering AWGN noise matrix.

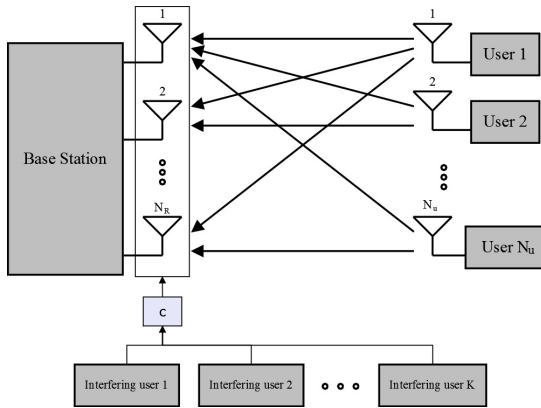


Fig. 2. Uplink MU-MIMO system model with pilot contamination.

Under these conditions the  $nr$ -th base station antenna receives the signal characterized by

$$\mathbf{Y}_{nr}^{PC} = \mathbf{Y}_{nr}^P + c(\mathbf{H}_{nr,1}^{PC}\mathbf{s}_1 + \mathbf{H}_{nr,2}^{PC}\mathbf{s}_2 + \dots + \mathbf{H}_{nr,K}^{PC}\mathbf{s}_K + \mathbf{Z}_{nr}^{PC}), \quad (10)$$

which is affected with inter-cell interference from  $K$  interfering users.

#### A. Iterative receiver and channel estimation

Consider the IB-DFE with the block diagram shown in figure 3. After discarding the CP samples, the equalizer's output at the  $i$ -th iteration for the  $k$ -th frequency is given

$$\tilde{\mathbf{X}}_k = \mathbf{F}_k^{(i)} \mathbf{Y}_k - \mathbf{B}_k^{(i)} \hat{\mathbf{X}}_k^{(i-1)}, \quad (11)$$

where  $\hat{\mathbf{X}}_k^{(i-1)}$  denotes the discrete Fourier transform (DFT) of the decision block from previous iteration, and the feed-forward  $\{\mathbf{F}_k; k = 0, 1, \dots, N-1\}$  and feedback  $\{\mathbf{B}_k; k = 0, 1, \dots, N-1\}$  coefficients are given by

$$\mathbf{F}_k = \frac{\kappa \mathbf{H}_k^H}{\alpha \mathbf{I} + (1 - \rho^2) \mathbf{H}_k \mathbf{H}_k^H}, \quad (12)$$

and

$$\mathbf{B}_k = \mathbf{F}_k \mathbf{H}_k - \mathbf{I}, \quad (13)$$

respectively.  $\rho$  denotes the correlation coefficient and gives a measure of the reliability of the decisions employed in the feedback loop. For each channel frequency  $\alpha = E[|N_k|^2]/E[|X_k|^2]$  and  $\kappa$  is selected so as to ensure that  $\sum_{k=0}^{N-1} \mathbf{F}_k \mathbf{H}_k / N = 1$ .

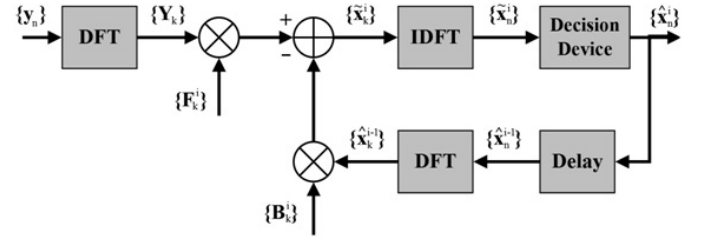


Fig. 3. IB-DFE receiver with soft decisions

Using this receiver CSI may be iteratively estimated as well. The process starts with the first iteration, where the CSI is estimated with the ZC sequences. For subsequent iterations (i.e.  $i > 1$ ), the frequency-domain estimated symbols of the previous iteration,  $\hat{X}_k^{i-1}$ , can be used, instead the pilots, to estimate the CSI more accurately. Using minimum square error (MSE) channel estimation technique, the channel estimate for the  $i$ th iteration is given by

$$\mathbf{H}_k^i = \left[ \left( \hat{X}_k^{i-1} \right)^H \hat{X}_k^{i-1} + \frac{\sigma_z^2}{\sigma_h^2} \mathbf{I} \right]^{-1} \left( \hat{X}_k^{i-1} \right)^H \mathbf{Y}_k. \quad (14)$$

The drawback of this process lies on the complexity associated to channel matrix inversions on (12). But this complexity is mainly associated to the data equalization and the channel estimation can be viewed as a subproduct of this iterative process (the channel estimation alone has a complexity of  $O(N_R N_u)$ ). However, the complexity associated to the IB-DFE can be a constrain when the number of channels involved grows, which is the case of a true mMIMO scenario. For that reason a simpler CSI estimation process with lower computational complexity like FSCAPI could be advantageous.

TABLE I. FSCAPI SUBSPACE TRACKING ALGORITHM [22]

|   |
|---|
| Initialization:<br>$\mathbf{W}(0) = \begin{bmatrix} \mathbf{I}_{N_u} \\ 0_{(N_R - N_u) \times N_u} \end{bmatrix}, \mathbf{Z}(0) = \mathbf{I}_{N_u}$   |
| For $n = 1, 2, \dots, (N_{data})$<br>Input vector: $\mathbf{y}(n)$<br>$\mathbf{r}(n) = \mathbf{W}(n-1)^H \mathbf{y}(n)$<br>$\mathbf{d}(n) = \mathbf{Z}(n-1) \mathbf{r}(n)$<br>$\mathbf{g}(n) = \frac{\mathbf{d}(n)}{\beta + \mathbf{r}(n)^H \mathbf{d}(n)}$<br>$e^2(n) = \ \mathbf{y}(n)\ ^2 - \ \mathbf{r}(n)\ ^2$<br>$s(n) = 1 + e^2(n) \ \mathbf{g}(n)\ ^2$<br>$\tau(n) = \frac{e^2(n)}{s(n) + \sqrt{s(n)}}$<br>$\phi'(n) = 1 - \tau(n) \ \mathbf{g}(n)\ ^2$<br>$\mathbf{r}'(n) = \phi(n) \mathbf{r}(n) + \tau(n) \mathbf{g}(n)$<br>$\mathbf{d}'(n) = \mathbf{Z}(n-1)^H \mathbf{r}'(n)$<br>$\mathbf{Z}(n) = \beta^{-1} (\mathbf{Z}(n-1) - \mathbf{g}(n) \mathbf{d}'(n)^H)$<br>$\mathbf{e}'(n) = \phi(n) \mathbf{y}(n) - \mathbf{W}(n-1) \mathbf{r}'(n)$<br>$\mathbf{W}(n) = \mathbf{W}(n-1) + \mathbf{e}'(n) \mathbf{g}(n)^H$<br>End for |

### B. FSCAPI channel estimation

To avoid high-complexity and very time consuming algorithms, in [16] and [17] were proposed some subspace tracking algorithms. The FSCAPI-based estimator is an adaptation of the fast approximated power iteration (FAPI) in [17]. The FSCAPI subspace tracking algorithm simplifies the iterative process of the correlation matrix, in order to resolve the ambiguity issue. With a complexity of  $O(N_R N_u)$  [22], it achieves lower computational complexity, albeit better tracking results.

The covariance matrix of the received signal can be given by

$$\mathbf{R}_y = E\{\mathbf{y}\mathbf{y}^H\} = E\{\mathbf{H}\mathbf{x}\mathbf{x}^H \mathbf{H}^H + \mathbf{z}\mathbf{z}^H\} = \mathbf{H}\mathbf{H}^H + \mathbf{I}_{N_R}. \quad (15)$$

To acknowledge the signal subspace, the covariance matrix  $\mathbf{R}_y$  can be decomposed using SVD such as

$$\mathbf{R}_y = [\mathbf{U}_s \mathbf{U}_n] \mathbf{\Lambda} [\mathbf{U}_s \mathbf{Y}_n], \quad (16)$$

where  $\mathbf{U}_n$  ( $\mathbf{U}_n \in \mathbb{C}^{N_R \times N_u}$ ) is the noise subspace and  $\mathbf{U}_s$  ( $\mathbf{U}_s \in \mathbb{C}^{N_R \times (N_R - N_u)}$ ) is the signal subspace. In [10] it was proven that  $\mathbf{U}_s$  determines the channel matrix  $\mathbf{H}$ , depending on a scalar multiplicative ambiguity matrix  $\mathbf{A}$  ( $\mathbf{A} \in \mathbb{C}^{N_u \times N_u}$ ). Therefore, the channel estimates can be given by

$$\hat{\mathbf{H}} = \mathbf{U}_s \mathbf{A}. \quad (17)$$

To calculate the ambiguity matrix, a short training sequence is used. According to [10], the ambiguity matrix can be calculated as

$$\mathbf{A} = (\mathbf{U}_s)^H \hat{\mathbf{H}}^P, \quad (18)$$

where  $\hat{\mathbf{H}}^P$  is the pilot-based channel estimate resulting of the least squares (LS) channel estimation. Therefore, the channel estimate  $\hat{\mathbf{H}}$  is given by

$$\hat{\mathbf{H}} = \mathbf{U}_s (\mathbf{U}_s)^H \hat{\mathbf{H}}^P. \quad (19)$$

The FSCAPI subspace tracking algorithm presented in Table 1, was proposed in [22] and was adopted due to its fast convergence and good tracking performance and it is presented in Table 1.

In the algorithm of Table 1,  $\mathbf{W}(n)$  ( $\mathbf{W}(n) \in \mathbb{C}^{N_R \times N_u}$ ) is the tracked signal subspace for the  $n$ -th sample,  $\beta$  ( $0 < \beta < 1$ ) is the forgetting factor and  $N_{data}$  ( $N_{data} = N - N_P$ ) denotes the length of the received signal without the pilots. The forgetting factor controls the influence of old data. Then, the tracked signal subspace  $\mathbf{W}(N_{data})$  is the estimation of  $\mathbf{U}_s$ , given by

$$\mathbf{U}_s = \mathbf{W}(N_{data}). \quad (20)$$

Therefore, the channel estimation matrix is given by

$$\hat{\mathbf{H}}^{FSCAPI} = \mathbf{W}(N_{data}) (\mathbf{W}(N_{data}))^H \hat{\mathbf{H}}^P. \quad (21)$$

In the following simulations based on this method, the selected forgetting factor is  $\beta = 0.996$ .

## IV. SIMULATION RESULTS

In all simulations, each user has  $N_a = 1$  antennas and the BS receives through  $N_R = N_u N_a$  antennas, where  $N_u$  denotes the number of active users. The users transmit simultaneously frame blocks with same sizes. Channel is assumed as time invariant over several transmitted blocks. It is also channel reciprocity between uplink and downlink. Regarding the pilot contamination, two different scenarios are considered: one without pilot contamination and another where the interference level due to pilot contamination is set at  $-10$  dB and  $-6$  dB due to the interference of  $K = 5$  users. The number of pilots may vary between  $N_P = 17$  and  $N_P = 47$  and 5 iterations are employed in the iterative channel estimation process based on the IB-DFE.

Monte Carlo experiments are used to obtain the average results of bit error rate (BER). The transmitted symbols  $x_n$  are selected with equal probability from a quadrature phase shift keying (QPSK) constellation. Time and phase synchronization at receiver are assumed as perfect. Also it is assumed linear power amplification at transmitter and receiver. The results are expressed as function of  $\frac{E_b}{N_0}$ , where  $N_0/2$  is the noise variance and  $E_b$  is the energy of the transmitted bits. The number of channels involved in the communication system is 1000, with  $N_R = 100$  receive antennas for  $N_u = 10$  users. It is important to mention that were considered 5 iterations for the IB-DFE based estimation method, because additional iterations do not have significant impact on the the system's performance.

In Fig. 4 are shown the BER results for both channel estimators, without pilot contamination. As expected, an higher number of pilots leads to a better BER performance. From the results it is perceptible the BER's improvement achieved by the iterative process associated to the IB-DFE. It can also be seen that the degradation of FSCAPI against the iterative process is less than 1 dB (for a BER value of  $10^{-4}$ ) for a number of pilots of  $N_P = 47$ . For a lower number of pilots the degradation is slightly higher with a value around 1.5 dB. Notwithstanding, the slight degradation of the FSCAPI, the lower complexity when compared with the iterative channel estimator could justify his choice when the number of user grows, independently of the number of pilots. In Fig. 5 it is shown the MSE evolution with SNR for a number of pilots of  $N_P = 17$  and  $N_P = 47$ . With  $N_P = 47$  both channels estimation methods have similar values for the MSE. As it can be seen from the results, only for  $N_P = 17$  the FSCAPI

presents a noticeable degradation face to the iterative estimator. This justifies the behavior of the BER results already presented in Fig. 4. Also, from these results it becomes clear that when the number of antennas increases the iterative channel estimation technique is not as effective as the FSCAPI at low SNRs.

In the following set of simulation results it is considered a mMIMO system with pilot contamination, composed by  $N_R = 100$  base station antennas,  $N_u = 10$  active users and  $K = 5$  interfering users correspond to the number of training sequences contaminated by an interfering channel attenuated by a factor  $c$  (with  $c_1 = -6\text{dB}$  and  $c_2 = -10\text{dB}$  as mentioned already). Fig. 6 shows the BER results for  $N_P = 17$  in presence of pilot contamination. It can be seen that the performances of both estimation methods have a similar behavior with the FSCAPI presenting a degradation lower than 1 dB. It is also visible the effect of the inter-cell interference on BER's performance for both channel estimators. Despite the clear effect of pilot contamination on system's performance, the behavior of MSE results could clarify which of the estimators will be the best choice for mMIMO systems with pilot contamination. Fig. 7 shows the MSE values for  $N_P = 17$  and  $N_P = 47$ , respectively. For a lower number of pilots, i.e.  $N_P = 17$ , the FSCAPI-based estimator outperforms the iterative one (note that even when  $N_P = 47$  the MSE performances are very similar). Having in mind these results, we may say that FSCAPI could be a better choice due to the linear computational complexity. It also benefits from large base station antennas arrays, since the subspace tracking algorithm relies on the orthogonality of channel matrix.

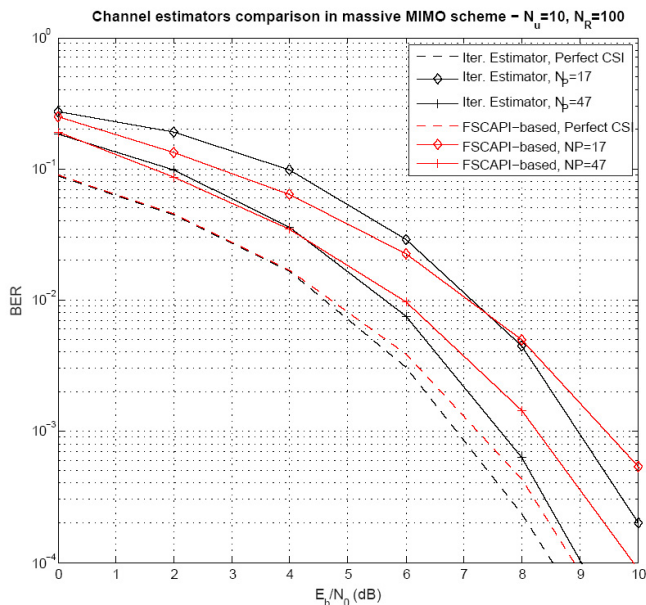


Fig. 4. BER performance comparison between channel estimation techniques, for  $N_u = 10$  and  $N_R = 100$ .

## V. CONCLUSIONS

It was shown how a two low complexity channel estimation schemes can be used to cope with pilot contamination in a multi-user mMIMO environment. Simulation results for

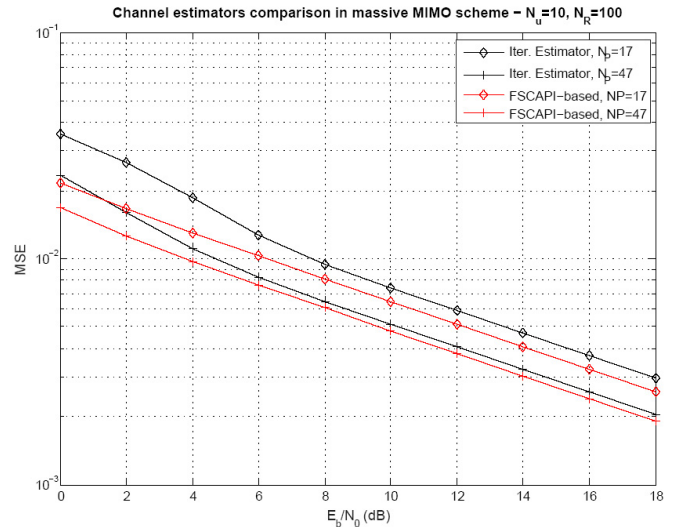


Fig. 5. MSE comparison between channel estimation techniques, for  $N_u = 10$  and  $N_R = 100$ .

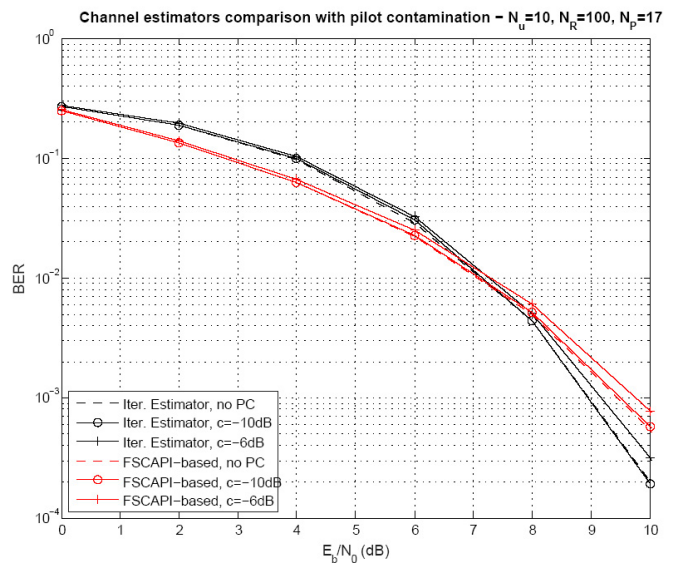


Fig. 6. BER comparison between channel estimation techniques in massive MIMO scheme with pilot contamination ( $c=-6\text{dB}$  and  $c=-10\text{dB}$ ) and  $N_u = 10$ ,  $N_R = 100$ , for  $N_P = 17$ .

both estimation methods demonstrate the ability to cope with pilot contamination. However, the FSCAPI-based estimator have similar MSE performance to the iterative one but with linear computational complexity. This makes the FSCAPI the obvious choice for mMIMO systems involving several hundreds of active channels. More important, this can be achieved without any sacrifice on spectral efficiency, since no additional pilots are needed.

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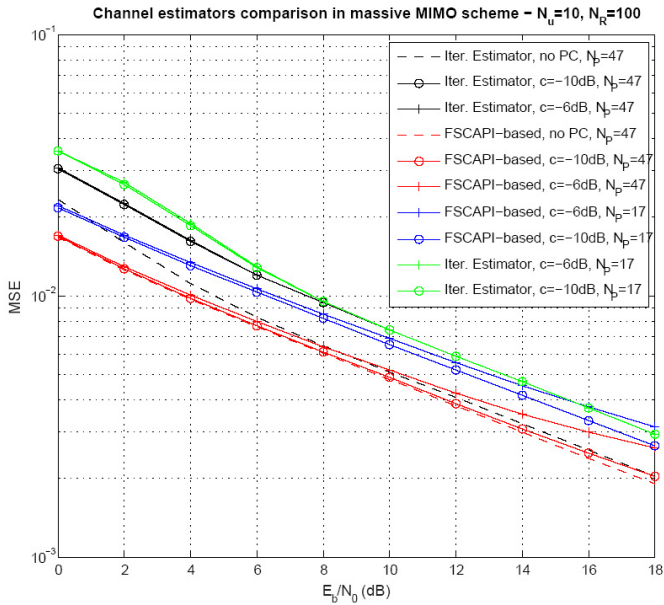


Fig. 7. MSE comparison between channel estimation techniques in massive MIMO scheme with pilot contamination ( $c=-6\text{dB}$  and  $c=-10\text{dB}$ ) and  $N_u = 10$ ,  $N_R = 100$ , for  $N_P = 17$  and  $N_P = 47$ .

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