A Mathematical Model of Crocodile Population

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Abstract – Crocodiles have been identified as a species which have indicated a strong survivorship over millions of years from pre-historic era. Sex of a crocodile hatchling is determined by incubation temperature of the egg during gestation. A nesting site of crocodiles is located in a river basin and it has been classified into three sub-regions according to the temperature variation which depends on the spatial distance from the water surface. Due to the low incubation temperature, all the hatchings of region (I) are females. In region (II), there is an equal likelihood to produce a male or female hatching and all male hatchings are produced in region (III). Following the criteria of Temperature-Dependent Sex Determination (TSD), a mathematical model of four first order non-autonomous differential equations, has been formulated based on a theoretical fraction of female population which incubate in region (I). In our approach, this theoretical fraction was modified in order to capture the dynamical evolution of the sex ratio more accurately than the outputs of existing models. Then, the carrying capacity term was also modified in order to analyze the dynamical evolution of the sex ratio under the influence of periodic flooding in the nesting sites. Finally, long term variation of the crocodile population was qualitatively interpreted using the numerical simulations of the model.

Keywords: Temperature-dependent sex determination; Differential equations; Sex ratio; Carrying capacity

I. INTRODUCTION

The influence of temperature-dependent sex determination (TSD) has been detected in all reptilian groups. Besides the genetic sex determination (GSD), TSD is highly observed among crocodiles [1],[2],[3]. In this scenario, male hatchlings are more likely to be produced under relatively high temperature values whereas more female off springs are born under low incubation temperature [2]. Temperature of a nesting site increases with the distance from the water surface, so the ratio of males to females hatched increases with an increasing distance from the river.

According to the temperature distribution of a river basin, three sub regions which indicate significant difference of the sex ratio of hatchlings.

- Region I (Wet marsh): All hatchings are females.
- Region II (Dry marsh): 50% of the produced hatchings are males where the others are females.
- Region III (Dry levees): All hatchings are males.

According ecological facts, existence of large number of female crocodiles assures a strong survivorship of the species. In [2], female population percentage of each region of a typical crocodile habitat is given as 79.6% for region (I), 13.6% for region II and 6.7% for region III. These figures have been used as carrying capacity values of each region where the influence of periodic floodings is not encountered.

The dynamical model of female and male population is preliminarily based on the theoretical fraction of female population which incubate in region (I). This fraction is defined as a function of carrying capacity of region (I) and the female population of it. In [3], it has stated that the fraction is not fully capable of capturing all the feature of variation. Therefore, a modified fraction has been alternatively defined satisfying all required conditions. It is observed that nesting sites of region (I) could be severely affected in periodic flood situations [1],[2],[3] and the periodic variation is also apparent in the carrying capacity of region (I). On this condition, carrying capacity term of region (I) was modified by inserting an experimentally estimated sinusoidal term [3]. Then, the variation of sex ratio was qualitatively interpreted by using the existing and modified approaches.

II. MATERIALS AND METHODS

It has been observed that only a fraction of females can incubate their eggs in the region (I). This fraction should be a function of the carrying capacity (k_1) and the female population (f_1) of region (I). In [2], it has been stated that fraction does not perfectly indicate the portion of female crocodiles incubate in region (I). Therefore, the fraction was modified as follows,

$$F = \frac{k_1^2}{k_1^2+f_1^2} \quad ...(1)$$

The modified fraction satisfies the necessary conditions, $F\rightarrow 0$ as $f_1\rightarrow\infty$ and $F\rightarrow 1$ as $f_1\rightarrow 0$. Then the new version of set of first order non-autonomous differential equations is defined using the same derivation procedure in [2],[3].

$$\frac{df_1}{dt} = b\left(\frac{k_1^2}{k_1^2+f_1^2}\right)f_1 - pf_1 \quad ...(2a)$$

$$\frac{df_2}{dt} = \frac{b}{2}\left(\frac{f_1^3}{k_1^2+f_1^2}+f_2\right)\left(\frac{k_2^2}{k_2^2+\left(\frac{f_1^3}{k_1^2+f_1^2}+f_2\right)^2}\right) - pf_2 \quad ...(2b)$$

$$\frac{dm_2}{dt} = \frac{b}{2}\left(\frac{f_1^3}{k_1^2+f_1^2}+f_2\right)\left(\frac{k_2^2}{k_2^2+\left(\frac{f_1^3}{k_1^2+f_1^2}+f_2\right)^2}\right) - pm_2 \quad ...(2c)$$

$$\frac{dm_3}{dt} = b\left(\frac{k_3^2}{k_3^2+\frac{\left(\frac{f_1^3}{k_1^2+f_1^2}+f_2\right)^6}{\left(k_2^2+\left(\frac{f_1^3}{k_1^2+f_1^2}+f_2\right)^2\right)^2}}\right)\left(\frac{\left(\frac{f_1^3}{k_1^2+f_1^2}+f_2\right)^3}{k_2^2+\left(\frac{f_1^3}{k_1^2+f_1^2}+f_2\right)^2}\right) - pm_3 \quad ...(2d)$$

In the equations (2a) - (2d), k_i $(i=1,2,3)$ denote the carrying capacity of each region I, II and III. f_1 and f_2 denote the population of female crocodiles of region I and II where m_2 and m_3 denote the male population of each region II and III. In addition, the effective birth rate and death rate are denoted by b and p respectively ($b>p$).

It can be shown that the system (2) is Lipchitz continuous with respect to the variables $f_1(t)$, $f_2(t)$, $m_2(t)$ and $m_3(t)$ and it confirms the existence of unique solutions of the system [3]. In order to represent the influence of periodic floods in the carrying capacity, k_1 is modified as follows,

$$k_1(t)=\begin{cases}\bar{k} & t\leq a\\ \bar{k}+A\sin\left(\frac{t-a}{T}\right) & t>a\end{cases} \quad ...(3)$$

In this case, the influence of periodic floods has been introduced after a years. The period of floodings (T) and amplitude (A) are experimentally estimated and $k_1(t)$ indicates its maximum value ($\bar{k}$) during the time without floods and minimum value is indicated at the peak of flood situation [2],[3].

It can be shown that the system (1) which is modified by incorporation of the term $k_1(t)$, has periodic solutions.

III. Results and discussions

In this work, a qualitative model was formulated to interpret the dynamics of sex ratio of crocodile population with the influence of periodic floodings. The formulated dynamical equations (2a) - (2d) and modified carrying capacity $k_1(t)$ were numerically simulated with experimental data in [2][3] using MATLAB@. In order to avoid the dependence of the solutions on initial conditions, the carrying capacity was kept constant up to 50 years and a flooding cycle of 63($\approx 20\pi$) years, was then incorporated in equation (3) along with experimental parameter values $\bar{k}=0.797$, $A=0.790$ [3]. Since the system (2) is locally asymptotically stable, its long-term dynamical evolution is independent of the initial conditions [3]. In this work, the qualitative model has been only developed to investigate the trends of population growth and it has not focused on specific ecological data. Therefore, the initial value of each model variable was chosen to be $f_1(0)=10$, $f_2(0)=10$, $m_2(0)=10$ and $m_3(0)=10$.

Confirming the influence of periodic floodings, female population of region I of figure 1(a), has indicated an oscillation with much greater amplitude than the population of region (II) and (III). Moreover, periods of total population variation of figure 1(e), modeled by both existing and modified approaches are equal to the corresponding periods indicated in figure 1(a). It should be noted that region (I) directly is affected by the floods and the population growth of other two regions is dependent on region (I).

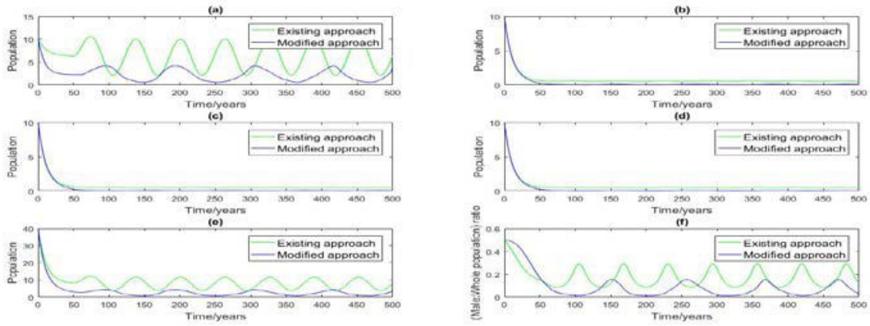

Figure 1. (a): Variation of female population of region I modelled by the existing approach and modified approach, (b): Variation of female population of region II modelled by the existing approach and modified approach, (c): Variation of male population of region II modelled by the existing approach and modified approach, (d): Variation of male population of region III modelled by the existing approach and modified approach, (e): Variation of whole population modelled by the existing approach and modified approach, (f) Dynamics of the sex ratio, Male population: whole population

According to the ecological interpretations in [1],[2], existence of high female crocodile population confirms an optimal growth of the population. Dynamical evolution of sex ratio given in figure 1(f) indicates the steady state sex ratio (number of males: number of females) of existing approach, is between 0.410 and 0.095 where it is between 0.180 and 0.010 in our modified approach. In [3], this average sex ratio measured based on experimental data is given as 0.12. Comparing two approaches in this work, our modified approach has given the average sex ratio to be 0.095 where its value of the existing approach is 0.25. Therefore, our new approach has been capable of approximating more accurate sex ratio variation than the existing approach.

IV. Conclusions

The modified theoretical fraction of females who incubate in region (I), has been used to get an accurate approximation for the steady state sex ratio of a crocodile population under the influence of periodic floodings. The qualitative outlook of the results, indicates how the crocodile population growth is naturally regulated with floodings. Numerical simulations of the derived dynamical system confirmed that it is locally asymptotically stable and the long-term evolution of the population is independent of the initial conditions. The model will be improved as a stochastic model by incorporating the randomness in variation of floodings.

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