DETERMINISTIC AND STOCHASTIC DYNAMICS OF LOTKA-VOLTERRA COMPETITION MODEL

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Abstract

The models of Continuous-time Markov chain (CTMC) and Ito stochastic differential equations (SDE) of population interactions based on a deterministic system have been proposed. The deterministic system is built upon the classical Lotka-Volterra competition system by incorporating Allee effects and stocking into one of the two populations. In this work, we use ordinary differential equations to model population interaction. Using the concept of uniform persistence, we derive sufficient conditions for coexistence of the two populations based on the boundary dynamics. The CTMC model is formulated first as a birth and death process and the derivation of the SDE model is then based on the CTMC. As these two stochastic models are time homogeneous, the transition probabilities are time independent. The transition probabilities of the CTMC satisfy the forward Kolmogorov differential equations and theoretically the moments of the random variables of the populations can be derived based on the moment generating functions of the transition probabilities. The Euler-Maruyama numerical method is applied to approximate the Ito stochastic differential model. Numerical simulations with parameter values taken from existing literature are performed to validate our analytical findings. We conclude that the coexistence of both competing populations under stocking is more robust as compared to the dynamical consequence of no stocking. Persistence occurs in the stochastic models when the populations are extinct in the deterministic setting. Therefore, stochastic modeling with Allee effects and stocking can alter competition outcomes significantly and has a profound effect on population interaction.

Keywords: Allee Effect, Stocking, Coexistence.